UNSTEADY PLANE-PARALLEL FLOW OF A PARTIALLY IONIZED GAS IN A STRONG MAGNETIC FIELD

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Larmor rotation of charged particles in a strong magnetic field leads to anisotropy of transport coefficients. The effect of anisotropy in conductivity (Hall effect) on a steady flow "of the Hartmann type" was taken into account for weakly ionized gas in [1 and 2], for completely ionized gas in [3] and for partially ionized gas in [4]. The effect of anisotropy in conductivity on unsteady flow has to our knowledge been investigated only for a weakly ionized gas [5 and 6]. Finally, the solution of the problem with consideration of anisotropy in viscosity (when the cyclotron frequency of rotation for ions is not small in comparison with their "collision" frequency) was obtained only in the steady case for a completely ionized gas [7].

On the basis of the system of equations obtained in [8] an unsteady plane-parallel flow of a partially ionized gas is examined below, taking into consideration the Hall effect, "slippage" of ions with respect to neutral particles, and anisotropy of viscosity. The medium in motion is presumed to be incompressible, the magnetic Reynolds number is assumed to be small. Thermal diffusion terms in Ohm's law and the temperature dependence of transport coefficients are neglected here. By means of Laplace transform an exact solution of the problem is obtained for the case of arbitrary time dependence of pressure gradient, and also for particular cases of pulsating and of constant pressure gradients.

1. The same model of a partially ionized gas moving in a strong external magnetic field, as was adopted in [8], is utilized here. Additional assumptions with respect to incompressibility of the medium (ρ = const), constancy of its degree of ionization (θ = const), and smallness of magnetic Reynolds number are introduced; then, if the effect of temperature on transport coefficients is neglected, they can be considered as constant quantities. Finally, neglecting the thermal diffusion effect we will have the following system of equations describing the motion of the medium to be examined [8]:

$$\operatorname{div} \mathbf{u} = 0, \qquad \operatorname{div} \mathbf{V}_i = 0, \qquad \rho\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\,\mathbf{u}\right] = -\nabla p - \operatorname{div} \boldsymbol{\pi} + \mathbf{j} \times \mathbf{B} \tag{1.1}$$

(1.1) cont.

$$\mathbf{j} + \frac{\omega_e \tau_0}{B}\left(\mathbf{j} \times \mathbf{B} - \frac{Zs}{1+Zs}\nabla p\right) + 2(1-s)^2 \frac{\omega_i \tau_{ia} \omega_e \tau_0}{B^2}[\mathbf{B} \times (\mathbf{j} \times \mathbf{B}) +$$
$$+ \frac{Zs}{1+Zs}\nabla p \times \mathbf{B} - \frac{1}{1-s}(s \operatorname{div} \pi - \operatorname{div} \pi_i) \times \mathbf{B}] = \sigma_0 (\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

$$\operatorname{rot} \frac{\mathbf{B}}{\mu_0} = j, \quad \operatorname{rot} \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{div} \mathbf{B} = 0, \quad \operatorname{div} \varepsilon_0 \mathbf{E} = \rho_e$$

$$\left(\frac{1}{\tau_0} = \frac{1}{\tau_{ei}} + \frac{1}{\tau_{ea}}\right)$$

Here π and π_i are viscosity tensors of the mixture and the ions, respectively

$$\pi^{rm} = -\eta^{(0)}W_0^{rm} - \eta^{(1)}W_1^{rm} - \eta^{(2)}W_2^{rm} + \eta^{(3)}W_3^{rm} + \eta^{(4)}W_4^{rm}$$
$$\pi_i^{rm} = -\eta_i^{(0)}W_0^{rm} - \eta_i^{(1)}W_1^{rm} - \eta_i^{(2)}W_2^{rm} + \eta_i^{(3)}W_3^{rm} + \eta_i^{(4)}W_4^{rm} \tag{1.2}$$

Expressions for viscosity coefficients $\eta^{(k)}$ and $\eta_i^{(k)}$ $(k = 0, 1, 2, 3, 4)$, and also for tensors W_k^{rm} are given in [8]. The velocity of "slippage" of ions with respect to neutral particles is given by Expression

$$\mathbf{V}_i = \frac{2\tau_{ia}}{s\rho}\left[\mathbf{j}\frac{B}{\omega_e \tau_{ea}} + (1-s)\,\mathbf{j} \times \mathbf{B} - \frac{Zs(1-s)}{1+Zs}\nabla p + s \operatorname{div} \pi - \operatorname{div} \pi_i\right] \tag{1.3}$$

Other notations are identical with those used in [8], namely: $\mathbf{u}$ is the average mass velocity, p is the pressure, $\mathbf{B}$ is the vector of magnetic induction, $\mathbf{j}$ is the vector of current density, $\mathbf{E}$ is the vector of electric field tension, Z is the charge number, ω_e and ω_i are cyclotron frequencies of electrons and ions, μ_0 and ε_0 are magnetic and electric constants, ρ_e is the space charge, $\sigma_0 =$ const is the conductivity, $\tau_{\alpha\beta}^{-1}$ is the effective collision frequency of particles of the α and β kind.

The system of equations written above is simplified considerably when the concrete problem of flow in a flat channel with the hight $2a$ is examined in the presence of homogeneous external fields $\mathbf{B}_0$ parallel to z and $\mathbf{E}_0$ perpendicular to z. The natural assumption that the velocity depends only on the transverse coordinate z and time t, and the possibility to neglect induced fields by virtue of the smallness of the magnetic Reynolds number, lead to a linearization of the original system of equations.

Taking into account that $u_z = j_z \equiv 0, B_z \equiv B_0 = \text{const}$ and introducing complex velocity $v(z,t)$, density of electrical current $J(zt)$, induced magnetic field $\chi(z,t)$, external electric field φ_0, eddy electric field $\varphi(z,t)$ and also the complex pressure gradient $\psi(t)$ according to the following equations

$$v = u_x - iu_y, \quad J = j_x - ij_y, \quad \chi = \mu_0^{-1}(B_x - iB_y) \tag{1.4}$$
$$\varphi_0 = E_{0x} - iE_{0y}, \quad \varphi = E_x - iE_y, \quad \psi = P_x - iP_y \tag{1.5}$$
$$P_x = -\partial p / \partial x, \qquad P_y = -\partial p / \partial y$$

we reduce the presented problem to solving a linear equation of the second order for $v(z,t)$

$$\rho\frac{\partial v}{\partial t}-\left[(\eta^{(2)}+i\eta^{(4)})\left(1+\frac{\delta s}{1-s}\xi\right)-(\eta_i^{(2)}+i\eta_i^{(4)})\frac{\delta}{1-s}\xi\right]\frac{\partial^2 v}{\partial z^2}+$$
$$+B_0^2\sigma_0\xi v=\left[1-\frac{Zs}{1+Zs}\xi(\delta+i\omega_e\tau_0)\right]\psi+iB_0\sigma_0\xi\varphi_0 \qquad (1.6)$$

where

$$\delta=2(1-s)^2\omega_i\tau_{ia}\omega_e\tau_0,\qquad \xi=(1+\delta+i\omega_e\tau_0)^{-1} \qquad (1.7)$$

Viscosity coefficients entering into (1.6) have the form

$$\eta^{(2)}+i\eta^{(4)}=\frac{\eta^{(0)}+{}^4/_9\,(\omega_i\tau_i\theta)^2\,\eta_a+i\,{}^2/_3\,\omega_i\tau_i\theta\,(\eta^{(0)}-\eta_a)}{1+{}^4/_9\,(\omega_i\tau_i\theta)^2}$$
$$\eta_i^{(2)}+i\eta_i^{(4)}=\frac{(1+i\,{}^2/_3\,\omega_i\tau_i\theta)\,\eta_i^{(0)}}{1+{}^4/_9\,(\omega_i\tau_i\theta)^2}\qquad\left(\omega_i=\frac{ZeB_0}{m_i}\right) \qquad (1.8)$$

Here Ze is the charge and m_i is the mass of the ion; expressions for viscosity coefficients of the entire mixture $\eta^{(0)}$, of ions $\eta_i^{(0)}$ and of "isolated" neutral particales η_a in the case when the magnetic field is equal to zero are given in [8]. $\tau_i\theta$ is related to the time between every possible kind of collision between ions; dimensionless parameters $\omega_i\tau_i\theta$ characterizes the anisotropy of viscosity coefficients.

It is interesting to note that with growing $\omega_i\tau_i\theta$ the coefficient of viscosity $\eta^{(2)}$ monotonously decreases from the value $\eta^{(0)}$ at $\omega_i\tau_i\theta=0$,, asymptotically approaching η_a for large values of $\omega_i\tau_i\theta$.

Analogously, $\eta_i^{(2)}$ falls off from $\eta_i^{(0)}$, approaching 0 at $\omega_i\tau_i\theta\gg 1$.

A different behavior is shown by $\eta^{(4)}$ and $\eta_i^{(4)}$: increasing from 0 at $\omega_i\tau_i\theta=0$, they reach a maximum at $\omega_i\tau_i\theta=1.5$

$$(\max\,[\eta^{(4)}/(\eta^0-\eta_a)]=\max\,\eta_i^{(4)}/\eta_i^{(0)}=0.5)$$

and then decrease monotonously to zero with increasing $\omega_i\tau_i\theta$.

After solution of Equation (1.6) with zero initial and boundary conditions, the complex current density J is determined from Formula

$$I=i\frac{\xi}{B_0}\left\{\frac{\delta}{1-s}\left[(\eta_i^{(2)}+i\eta_i^{(4)})-s\,(\eta^{(2)}+i\eta^{(4)})\right]\frac{\partial^2 v}{\partial z^2}+\right.$$
$$\left.+B_0^2\sigma_0 v+\frac{Zs}{1+Zs}(\delta+i\omega_e\tau_0)\,\psi-iB_0\sigma_0\varphi_0\right\} \qquad (1.9)$$

The induced electric and magnetic fields are given in the first approximation by Equations

$$\frac{\partial\chi}{\partial z}=iJ,\qquad \mu_0\frac{\partial\chi}{\partial t}=i\frac{\partial\varphi}{\partial z} \qquad (1.10)$$

with corresponding initial and boundary conditions. From the projection of the equation of motion on z-axis and from the generalized form of Ohm's law, $\partial p / \partial z$, E_z and $\rho_e = \varepsilon_0 \partial E_z / \partial z$ can also be determined.

2. Introducing U_0, a, a / U_0, $\rho U_0^2 / a$ and $B_0 U_0$, as scalar quantities for v, z, t, ψ and φ_0, respectively, we rewrite Equation (1.6) in the dimensionless form

$$\frac{\partial v}{\partial t} - \left[\frac{1}{R^*}\left(1 + \frac{\delta s}{1-s}\xi\right) - \frac{1}{R_i^*}\frac{\delta}{1-s}\xi\right]\frac{\partial^2 v}{\partial z^2} + N^* v =$$

$$= \left[1 - \frac{Zs}{1+Zs}\,\xi\,(\delta + i\omega_e\tau_0)\right]\psi + iN^*\varphi_0 \tag{2.1}$$

In the equality (2.1) complex nondimensional Reynolds numbers R^* and R_i^*, and the parameter of magnetic interaction N^* are separated out

$$\frac{1}{R^*} = \frac{1}{R^{(2)}} + i\frac{1}{R^{(4)}}, \qquad \frac{1}{R_i^*} = \frac{1}{R_i^{(2)}} + i\frac{1}{R_i^{(4)}}, \qquad N^* = N\xi \tag{2.2}$$

where

$$R^{(k)} = \frac{U_0 a \rho}{\eta^{(k)}}, \qquad R_i^{(k)} = \frac{U_0 a \rho}{\eta_i^{(k)}} \qquad (k = 2.4) \tag{2.3}$$

$$N = \frac{B_0^2 \sigma_0 a}{\rho U_0} = S R_m, \quad S = \frac{B_0^2}{\mu_0 \rho U_0^2}, \qquad R_m = \mu_0 \sigma_0 a U_0 \tag{2.4}$$

Applying Laplace transformation to (2.1)

$$F(z, \beta) = \int_0^\infty f(z, t)\, e^{-\beta t}\, dt \tag{2.5}$$

and taking into account the initial condition

$$v(z, 0) = 0 \tag{2.6}$$

we obtain the following equation for representing $V(z, \beta)$:

$$-\frac{1}{R_\delta^*}V'' + (\beta + N^*)V = D(\beta) \tag{2.7}$$

where

$$\frac{1}{R_\delta^*} = \frac{1}{R^*}\left(1 + \frac{\delta s}{1-s}\,\xi\right) - \frac{1}{R_i^*}\frac{\delta}{1-s}\xi \tag{2.8}$$

$$D(\beta) = \left[1 - \frac{Zs}{1+Zs}\xi\,(\delta + i\omega_e\tau_0)\right]\Psi(\beta) + iN^*\frac{\varphi_0}{\beta} \tag{2.9}$$

Taking into account the boundary condition

$$V(\pm 1, \beta) = 0 \tag{2.10}$$

we find the solution for Equation (2.7)

$$V(z, \beta) = \frac{D(\beta)}{\beta + N^*}\left[1 - \frac{\cosh z\sqrt{R_\delta^*(\beta + N^*)}}{\cosh\sqrt{R_\delta^*(\beta + N^*)}}\right] \tag{2.11}$$

The Rieman-Mellin transformation theorem gives

$$v(z,\ t)=\frac{1}{2\pi i}\int\limits_{b-i\infty}^{b+i\infty}\frac{D(\beta)}{\beta+N^*}\left[1-\frac{\cosh z\sqrt{R_\delta^*(\beta+N^*)}}{\cosh\sqrt{R_\delta^*(\beta+N^*)}}\right]e^{\beta t}\,d\beta \qquad (2.12)$$

Utilizing the convolution theorem, Equation (2.12) can be rewritten in the form

$$v(z,\ t)=\int\limits_0^t\left\{\left[1-\frac{Zs}{1+Zs}\,\xi\,(\delta+i\omega_e\tau_0)\right]\psi(t-\tau)+\right.$$

$$\left.+\,iN^*\varphi_0\right\}\frac{1}{2\pi i}\int\limits_{b-i\infty}^{b+i\infty}\left[1-\frac{\cosh z\sqrt{R_\delta^*(\beta+N^*)}}{\cosh\sqrt{R_\delta^*(\beta+N^*)}}\right]\frac{e^{\beta\tau}}{\beta+N^*}\,d\beta d\tau \qquad (2.13)$$

The inside integral in (2.13) is easily found with the aid of the residue theorem, and the general solution at arbitrary pressure gradients $\psi(t)$ finally takes the form

$$v(z,\ t)=2\int\limits_0^t\left\{\left[1-\frac{Zs}{1+Zs}\,\xi\,(\delta+i\omega_e\tau_0)\right]\psi(t-\tau)+\right.$$

$$\left.+\,iN^*\varphi_0\right\}\exp(-N^*\tau)\sum_{k=0}^{\infty}(-1)^k\,\frac{\cos z\lambda_k}{\lambda_k}\exp\frac{-\lambda_k^2\tau}{R_\delta^*}\,d\tau \qquad (2.14)$$

where

$$\lambda_k=\frac{2k+1}{2}\pi \qquad (k=0,1,2,\ldots) \qquad (2.15)$$

3. We will now examine the case of pulsating pressure gradients. Let ν be the nondimensional cyclic frequency of the forcing pressure gradient. In (2.14) we write $\psi(t)=\psi_0\cos\nu t$. Carrying out the integration and summing the trigonometric series entering into the stationary state, a solution is obtained in the form

$$v(z,\ t)=\frac{\psi_0}{(N^{*2}+\nu^2)A^2}\left[1-\frac{Zs}{1+Zs}\,\xi\,(\delta+i\omega_e\tau_0)\right]\left[(N^*\cos\nu t+\right.$$

$$+\,\nu\sin\nu t)(A^2-\cosh zr_1\cosh r_1\cos zr_2\cos r_2-\sinh zr_1\sinh r_1\sin zr_2\sin r_2)+$$

$$\left.+\,(N^*\sin\nu t-\nu\cos\nu t)(\sinh zr_1\cosh r_1\sin zr_2\cos r_2-\cosh zr_1\sinh r_1\cos zr_2\sin r_2)\right]+$$

$$+\,i\varphi_0\left(1-\frac{\cosh z\sqrt{R_\delta^*N^*}}{\cosh\sqrt{R_\delta^*N^*}}\right)+2R_\delta^*\exp(-N^*t)\left\{\left[1-\frac{Zs}{1+Zs}\,\xi\,(\delta+i\omega_e\tau_0)\right]\times\right.$$

$$\times\,\psi_0\sum_{k=0}^{\infty}(-1)^{k+1}\frac{(\lambda_k^2+R_\delta^*N^*)\cos z\lambda_k}{\lambda_k\left[(\lambda_k^2+R_\delta^*N^*)^2+R_\delta^{*2}\nu^2\right]}\exp\frac{-\lambda_k^2 t}{R_\delta^*}+$$

$$\left.+\,iN^*\varphi_0\sum_{k=0}^{\infty}(-1)^{k+1}\frac{\cos z\lambda_k}{\lambda_k(\lambda_k^2+R_\delta^*N^*)}\exp\frac{-\lambda_k^2 t}{R_\delta^*}\right\} \qquad (3.1)$$

where

$$A = \cosh^2 r_1 \cos^2 r_2 + \sinh^2 r_1 \sin^2 r_2 \tag{3.2}$$

$$r_{1,2} = \sqrt{{}^1/_2 R_\delta^*(\sqrt{N^{*2} + \nu^2} \pm N^*)} \tag{3.3}$$

Series which enter into (3.1) represent a transition state which can be realized in the form of decaying harmonic oscillations with time, when the driving force does not change the frequency of these oscillations but only influences their amplitude. The other components give the stationary state which in particular contains forced oscillations with the frequency ν.

It is interesting to note that anisotropy in conductivity permits the attainment of maximum amplitudes of forced oscillations through the selection of frequency ν.

For the sake of simplicity we will demonstrate this for the case of inviscid medium where solution (3.1) acquires the form

$$v(z, t) = \frac{\psi_0}{N^{*2} + \nu^2}\left[1 - \frac{Zs}{1 + Zs}\,\xi\,(\delta + i\omega_e\tau_e)\right](N^* \cos \nu t + \nu \sin \nu t) +$$

$$+ i\varphi_0 - \left\{\frac{\psi_0 N^*}{N^{*2} + \nu}\left[1 - \frac{Zs}{1 + Zs}\,\xi\,(\delta + i\omega_e\tau_e)\right] + i\varphi_0\right\}\exp(-N^* t) \tag{3.4}$$

For the purpose of further simplification we will consider the gas to be weakly ionized ($s \ll 1$) and the external electric field to be absent ($\varphi_0 = 0$). Then, for the longitudinal velocity of the stationary state (ψ_0 is a real number) we obtain Expression

$$\overset{\circ}{u}_x(z, t) = \sqrt{f_1^2 + f_2^2}\cos\left(\nu t - \tan^{-1}\frac{f_1}{f_2}\right) \tag{3.5}$$

where

$$f_1 = \frac{\psi_0\nu\,(\alpha^2 - \beta^2 + \nu^2)}{(\alpha^2 - \beta^2 + \nu^2)^2 + 4\alpha^2\beta^2}, \qquad f_2 = \frac{\psi_0\alpha\,(\alpha^2 + \beta^2 + \nu^2)}{(\alpha^2 - \beta^2 + \nu^2)^2 + 4\alpha^2\beta^2} \tag{3.6}$$

$$\alpha = \frac{N\,(1 + 2\omega_i\tau_{ia}\omega_e\tau_{ea})}{(1 + 2\omega_i\tau_{ia}\omega_e\tau_{ea})^2 + (\omega_e\tau_{ea})^2}, \qquad \beta = \frac{N\omega_e\tau_{ea}}{(1 + 2\omega_i\tau_{ia}\omega_e\tau_{ea})^2 + (\omega_e\tau_{ea})^2} \tag{3.7}$$

Examining the amplitude $\sqrt{f_1^2 + f_2^2}$ for an extremum, we find that the maximum is attained at

$$\nu^2 = \alpha^2\left(\sqrt{\varepsilon^4 + 4\varepsilon^2} - 1\right) \qquad \left(\varepsilon = \frac{\beta}{\alpha} = \frac{\omega_e\tau_{ea}}{1 + 2\omega_i\tau_{ia}\omega_e\tau_{ea}}\right) \tag{3.8}$$

From (3.8) it follows that the indicated maximum can occur only for

$$\varepsilon \geqslant \sqrt{\sqrt{5} - 2} \approx 0.486 \tag{3.9}$$

Which in the present case represents the anisotropy in conductivity

In the isotropic case, however, when $\omega_e\tau_{ea} \ll 1$ and $\varepsilon \ll 1$, the amplitude has the form

$$\sqrt{f_1^2 + f_2^2} = \frac{\psi_0}{\sqrt{N^2 + \nu^2}} \tag{3.10}$$

i.e. it appears as a monotonously decreasing function of frequency ν.

It is also noted that the presence of parameter $\omega_i\tau_{ia}$ in the denominator of Expression (3.8) limits from above the magnitude of magnetic field for which (3.9) is correct

$$\omega_i\tau_{ia} < \frac{1}{2\sqrt{\sqrt{5}-2}} \qquad (3.11)$$

i.e. for sufficiently strong magnetic fields the effect of ion "slippage" with respect to neutral particles masks the effect due to the presence of external frequency. Analogously, for transverse (Hall) stationary velocity u_y° we will have

$$u_y^\circ = -\sqrt{f_3^2 + f_4^2}\cos\left(\nu t - \tan^{-1}\frac{f_3}{f_4}\right) \qquad (3.12)$$

where

$$f_3 = \beta\psi_0\frac{2\alpha\nu}{(\alpha^2-\beta^2+\nu^2)^2+4\alpha^2\beta^2}, \qquad f_4 = \beta\psi_0\frac{\alpha^2+\beta^2-\nu^2}{(\alpha^2-\beta^2+\nu^2)^2+4\alpha^2\beta^2} \qquad (3.13)$$

Here the maximum in amplitude is reached at

$$\nu^2 = \alpha^2(\varepsilon^2 - 1) \qquad (3.14)$$

which is meaningful for

$$\varepsilon \geqslant 1, \qquad \omega_i\tau_{ia} < 1/2 \qquad (3.15)$$

4. Let us examine the case of constant pressure gradients. Assuming that $\nu = 0$, we obtain from (3.1) the solution of the problem for the case of constant pressure gradients $\psi = \psi_0$

$$v(z,t) = \frac{\Delta}{N^*}\left(1 - \frac{\cosh zM_\delta^*}{\cosh M_\delta^*}\right) +$$

$$+ 2R_\delta^*\Delta\exp(-N^*t)\sum_{k=0}^{\infty}\frac{(-1)^{k+1}\cos z\lambda_k}{\lambda_k(\lambda_k^2+M_\delta^{*2})}\exp\frac{-\lambda_k^2}{R_\delta^*}t \qquad (4.1)$$

Here

$$\Delta = \left[1 - \frac{Zs}{1+Zs}\xi(\delta + i\omega_e\tau_0)\right]\psi_0 + iN^*\varphi_0 \qquad (4.2)$$

$$M_\delta^{*2} = R_\delta^*N^* = \xi M^{*2}M_i^{*2}\left[M_i^{*2}\left(1 + \frac{\delta s}{1-s}\xi\right) - M^{*2}\frac{\delta}{1-s}\xi\right]^{-1} \qquad (4.3)$$

Complex Hartmann numbers M^* and M_i^* are given by Equations

$$\frac{1}{M^{*2}} = \frac{1}{R^*N} = \frac{1}{(M^{(2)})^2} + i\frac{1}{(M^{(4)})^2},$$

$$\frac{1}{M_i^{*2}} = \frac{1}{R_i^*N} = \frac{1}{(M_i^{(2)})^2} + i\frac{1}{(M_i^{(4)})^2} \qquad (4.4)$$

where

$$M^{(k)} = \sqrt{R^{(k)}N} = B_0a\left(\frac{\sigma_0}{\eta^{(k)}}\right)^{1/2} \qquad (4.5)$$

$$M_i^{(k)} = \sqrt{R_i^{(k)}N} = B_0a\left(\frac{\sigma_0}{\eta_i^{(k)}}\right)^{1/2} \qquad (k = 2, 4)$$

The first term in (4.1) corresponds to the stationary state of the flow under investigation. This state represents harmonic oscillations along the coordinate because of the joint influence of viscosity and anisotropy in conductivity.

The decaying harmonic oscillation with time represents the transition state which corresponds to the second term in (4.1); here the cyclic frequency of these oscillations has the form

$$\Omega_k = \frac{N\omega_e\tau_0}{(1+\delta)^2+(\omega_e\tau_0)^2} - \lambda_k^2 \left\{ \frac{1}{R^{(4)}} + \frac{\delta s}{(1-s)\left[(1+\delta)^2+(\omega_e\tau_0)^2\right]} \times \right.$$
$$\left. \times \left[(1+\delta)\left(\frac{1}{R^{(4)}} - \frac{1}{sR_i^{(4)}}\right) - \omega_e\tau_0\left(\frac{1}{R^{(2)}} - \frac{1}{sR_i^{(2)}}\right)\right]\right\} \qquad (4.6)$$

We will examine now some specific cases of the solution which was found.

1. In the absence of anisotropy in conductivity $(\omega_e\tau_0 \ll 1)$ we will have for the longitudinal velocity component u_x the ordinary Hartmann stationary state and a transition state aperiodic with time. Here the transverse velocity component $u_y \equiv 0$, if $P_y = E_{0x} \equiv 0$.

2. If there is anisotropy in conductivity but anisotropy in viscosity is absent, then it is appropriate to write $\omega_i\tau_i\theta \ll 1$ and $\omega_i\tau_{ia} \ll 1$. in the solution. In addition, considering the gas to be weakly ionized $(s \ll 1)$ and the external electrical field to be absent $(\varphi_0 = 0)$, we will obtain the expression for velocity which coincides with the one derived in [6].

We note also the anisotropy in conductivity leads to a transition state which is periodic with respect to time and has the cyclic frequency

$$\Omega = N\frac{\omega_e\tau_0}{1+(\omega_e\tau_0)^2} \qquad (4.7)$$

In the case of a nonviscous medium for $\omega_i\tau_{ia} \ll 1$ (absence of "slippage" of ions) we obtain from (3.4)

$$v(z,t) = \left[\frac{\psi_0}{N}\left(1 + i\frac{\omega_e\tau_0}{1+Zs}\right) + i\varphi_0\right]\left[1 - \exp(-N^*t)\right] \qquad (4.8)$$

Assuming that $\psi_0 = P_x$ and $\varphi_0 = 0$, we have for velocities u_x° and u_y° of the stationary state

$$u_x^\circ = \frac{P_x}{N}, \qquad u_y^\circ = -\frac{P_x}{N}\frac{\omega_e\tau_0}{1+Zs} \qquad (4.9)$$

From this it can be seen that with increasing degree of ionization s, there is a decreasing of u_x° at the expense of corresponding increase in conductivity σ_0. There is also a more effective decrease in Hall velocity u_y° which is related to an increase in electrical current and consequently to an increase in retarding action of the ponderomotive force.

3. Consideration of effects of anisotropy of viscosity and of "slippage" of ions with respect to neutral particles leads to a considerable complication of the flow pattern. In particular, as is evident from (4.6), the presence anisotropy in viscosity leads to a spectrum of frequencies for the transitional state instead of one frequency (4.7) for $\omega_i\tau_i\theta \ll 1$. For instance, for completely ionized gas $(s = 1)$

$$\Omega_k = \frac{N\omega_e\tau_{ei}}{1+(\omega_e\tau_{ei})^2} - \lambda_k^2 \frac{1}{R_i^{(4)}} \tag{4.10}$$

We also note that the stationary state for fully ionized gas which can be obtained from (4.1) coincides with results from [7]. Finally, taking into account "slippage" of ions in Equations (4.9) gives

$$u_x^\circ = \frac{P_x}{N}\left[1+\frac{2(1-s)^2\,\omega_i\tau_{ia}\omega_e\tau_0}{1+Zs}\right], \qquad u_y^\circ = -\frac{P_x}{N}\frac{\omega_e\tau_0}{1+Zs} \tag{4.11}$$

From this it is evident that "slippage" of ions does not have an influence on the Hall velocity of the stationary state, but for a given $s \neq 1$ it increases the longitudinal component of velocity.

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